Algorithms for two-agent scheduling with earliness cost

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Abstract. The paper considers a two-agent scheduling problem on a single-machine.In the problem we have two agents and each agent has his own job set and the cost functions of his jobs. The goal of the problem is to minimize the weighted sum of the objective functions of the two agents. In this paper, we propose polynimal time algorithms for the problem.

Key words. Scheduling, Two-agent, Earliness, Optimal.

1. Introduction

Multi-agent scheduling was first introduced by Agnetis et al. [1] and Baker and Smith [2]. In the context of multi-agent scheduling, there are two or more agents and each agent has his own objective function. In the scope of multi-agent scheduling, much work has been done in recent years. Baker and Smith [2] focused on minimizing the linear combination of the objective functions of the two or more agents on a single machine. They examine the implications of minimizing an aggregate scheduling objective function in which jobs belonging to different agents are evaluated based on their individual criteria. Agnetis et al. [1] considered the scheduling models in which two agents compete for the usage of shared processing resource and each agent has his own criterion to optimize. In the scope of multi-agent scheduling, much work has been done in recent years. Agnetis et al. [3] studied the single-machine scheduling with multiple agents. Cheng et al. [4,5] also studied multi-agent scheduling on a single machine to minimize the total number of tardy jobs and the max-form objectives. Leung et al. [6] considered the identical parallel-machine scheduling problems with two agents, where the jobs of the two agents may have distinct release

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dates and preemption is allowed. B. Mor and G. Mosheiov [7] researched scheduling problems with two competing agents to minimize minmax and minsum earliness measures. Feng et al. [8] considered two-agent scheduling problems with rejection on a single machine. Other More related results can see [9,10].

In the problem, there are two competing agents A and B, each of them has a set of non-preemptive jobs to be processed on a single machine. Agent $x, x \in \{A, B\}$ has to process the job set. For convenience, we simply refer to a job of agent x as an x-job. We assume that the processing times and due dates are integers. Let θ indicate a feasible schedule of the $n = n_A + n_B$ jobs, i.e., a feasible assignment of starting times to the jobs of two agents. Write $P = \sum_{i=1}^{n_A} p_i^A + \sum_{i=1}^{n_B} p_i^B$.

As commonly assumed in scheduling problems with earliness objective, we restrict all jobs to be completed before a common deadline. Thus, let D denote the common deadline of all jobs. Note that, in order to guarantee a schedule is feasible, D must satisfy $D \geq P$.

The following notation are used throughout this paper:

p_j^x is the processing time of job J_j^x, $x \in \{A, B\}$. C_j^x is the completion time of job J_j^x, $x \in \{A, B\}$. d_j^x is the due date of job J_j^x, $x \in \{A, B\}$. $E_j^x = \max\{0, d_j^x - C_j^x\}$ is the earliness of the x-jobs, $x \in \{A, B\}$. $E_{\max}^x = \max_{J_j^x \in \text{Im}^x}\{E_j^x\}$ is the maximum earliness of x-jobs, $x \in \{A, B\}$. $\sum E_j^x$ is the total earliness of the x-jobs.

2. Paper contents

In the problem, agent A has objective γ^A, where $\gamma^A \in \{E_{\max}^A, \sum E_j^A\}$. Agent B has objective $E_{\max}^B$ which is the maximum earliness of his jobs. The goal is to find an optimal schedule such that $\gamma^A + \alpha E_{\max}^B$ is minimized, where $\alpha > 0$. For this problem, a polynomial-time algorithm is proposed. Note that since both agents try to minimize earliness, it is clear that jobs will be processed as late as possible. Therefore, an optimal schedule exists such that the first job starts at time $s = D - P$ and there is no idle times between consecutive jobs. Throughout this paper, we restrict our attention to the schedules with this property.

If a job J_j^x starts at time s_j^x, then it is completed at time $s_j^x + p_j^x$. The earliness of J_j^x is $E_j^x = \max\{0, d_j^x - p_j^x\}$. The following properties is established in B. Mor and G. Mosheiov [7]. We state each result from the perspective of agent A, but these properties also hold for agent B.

Property 1 ([7]) If agent A takes the $E_{\max}^B$ objective, then there is an optimal schedule in which all jobs belonging to are processed in non-decreasing order of $d_j^A - p_j^A$.

Property 2 ([7]) If all jobs of agent A have identical due dates and takes the $\sum E_j^A$ objective, then there is an optimal schedule in which all jobs belonging to J^A are processed in non-increasing order of p_j^A.

It is said a schedule σ for $1 \| \gamma^A + \alpha E_{\max}^B$, $\gamma^A \in \{E_{\max}^A, \sum E_j^A\}$ is effective if σ satisfies the following conditions.

(1) σ sequences jobs in J^B in non-decreasing order of $d_j^B - p_j^B$.

(2) σ sequences jobs in J^A in non-decreasing order of $d_j^A - p_j^A$ when $\gamma^A = E_{\max}^A$.

If all jobs of agent A have identical due dates, then σ sequences jobs in J^A in non-increasing order of p_j^A when $\gamma^A = \sum E_j^A$.

In the following we only consider the effective schedules. Thus, the jobs are re-labeled in J^B such that $d_1^B - p_1^B \leq \cdots \leq d_{n_B}^B - p_{n_B}^B$. Furthermore, if $\gamma^A = E_{\max}^A$, the jobs in J^A are re-labeled such that $d_1^B - p_1^B \leq \cdots \leq d_{n_B}^B - p_{n_B}^B$. If all jobs of agent A have identical due dates, and $\gamma^A = \sum E_j^A$, the jobs are re-labeled in J^A such that $p_1^A \geq \cdots \geq p_{n_A}^A$.

Theorem 1. For $\gamma^A = E_{\max}^A$, if all jobs of agent A have identical due date d, problem $1 \left\| \gamma^A + \alpha E_{\max}^B \right.$ can be solved in $O\left(n_A n_B \left(n_A + n_B\right)\right)$ time.

Proof. Let $\sigma_1 = \left\{J_1^A, \ldots J_{n_A}^A, J_1^B, \ldots J_{n_B}^B\right\}$ and $\sigma_2 = \left\{J_1^B, \ldots J_{n_B}^B, J_1^A, \ldots J_{n_A}^A\right\}$. Define $LB = E_{\max}^B\left(\sigma_1\right)$ and $UB = E_{\max}^B\left(\sigma_2\right)$. Then, for any effective schedule σ for $1 \left\| \gamma^A + \alpha E_{\max}^B \right.$, it must have $LB \leq E_{\max}^B \leq UB$. For $0 \leq u \leq n_A, 0 \leq v \leq n_B$, write

$t\left(u, v\right) = s + \sum_{0 \leq i \leq u} p_i^A + \sum_{0 \leq i \leq v} p_i^B$, where s is the start time of the first job on the machine. Define $y\left(u, v\right) = \max\left\{0, d_v^B - t\left(u, v\right)\right\}$. By noting that, in any effective schedule σ, the completion time of each job J_v^B must be of the form $t\left(u, v\right)$ for some u with $0 \leq u \leq n_A$, we must have $E_{\max}^B\left(\sigma\right) \in \left\{y\left(u, v\right) : 0 \leq u \leq n_A, 0 \leq v \leq n_B\right\}$.

For each $y \in \left\{y\left(u, v\right) : 0 \leq u \leq n_A, 0 \leq v \leq n_B\right\}$ with, the problem $1 \left\| \gamma^A + \alpha E_{\max}^B \right.$ is considered under the restriction that $E_{\max}^B = y$. The restricted version will be denoted by $1 \left| E_{\max}^B = y \right| \gamma^A$. There may be some y such that the problem $1 \left| E_{\max}^B = y \right| \gamma^A$ is infeasible. Hence, It is prefered to consider the relaxed version $1 \left| E_{\max}^B \leq y \right| \gamma^A$.

In an optimal effective schedule σ for $1 \left| E_{\max}^B \leq y \right| \gamma^A$, suppose (by the effective property of σ) that the set of the first $u + v$ jobs is $\sigma_1 = \left\{J_1^A, \ldots J_u^A, J_1^B, \ldots J_v^B\right\}$. If $E_{\max}^B > y$, then the $u + v$-th job under σ must be a job in J^A with processing time p_u^A. If $E_{\max}^B \leq y$, then the $u + v$-th job under σ must be J_v^B. Consequently, the problem $1 \left| E_{\max}^B \leq y \right| \gamma^A$ can be solved by the following algorithm.

Algorithm for $1 \left| E_{\max}^B \leq y \right| \gamma^A$

Step 1. Set $u = n_A, v = n_2$ and $F = 0$ for $\gamma^A \in \left\{E_{\max}^A, \sum E_j^A\right\}$.

Step 2. If $u = 0$, then define $\pi\left(i\right) = p_i^B, 1 \leq i \leq v$ and stop.

Denote by F_y the F-value returned by the above algorithm for a given y. The final observation is that the optimal objective valve of the problem $1 \left\| \gamma^A + \alpha E_{\max}^B \right.$ must be $\min\left\{F_y + \alpha y : LB \leq y \leq UB, y \in \left\{y\left(u, v\right) : 0 \leq u \leq n_A, 0 \leq v \leq n_B\right\}\right\}$. Since the complexity of the algorithm for $1 \left| E_{\max}^B \leq y \right| \gamma^A$ is $O\left(n_A + n_B\right)$ and It has at most $n_A n_B$ choices for y, therefore the problem can be solved in $O\left(n_A n_B \left(n_A + n_B\right)\right)$ time.

3. Conclusions

The paper considers a two-agent scheduling problem on a single-machine. The objective of the problem is to minimize the weighted sum of the objective functions of the two agents. In this paper, a polynimal- time algorithm for the problem is provided. For the further research, it is interesting to resolve the complexity of more than two agents.

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